Lecture 3

## Chapter 2

## Composition and Resolution of Concurrent forces

### Resultant Force :-

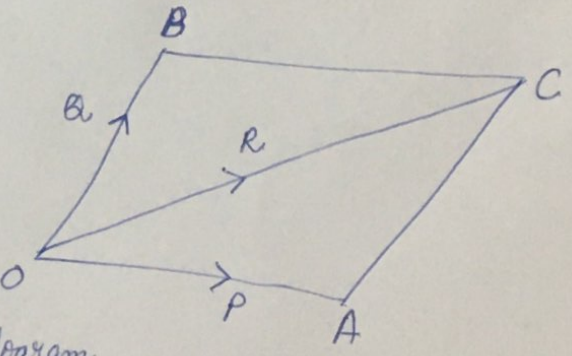
If two or more forces act on a rigid body (particle) and there exists a single force whose effect on the rigid body (particle) is same as of given forces, then this single force is called "resultant" of the given forces.

The given forces are called "components" of the resultant force.

**Existence :-**

It may be noted that it is not always possible to find a single force that is resultant of a given system of forces.

### Parallelogram Law of Forces :-



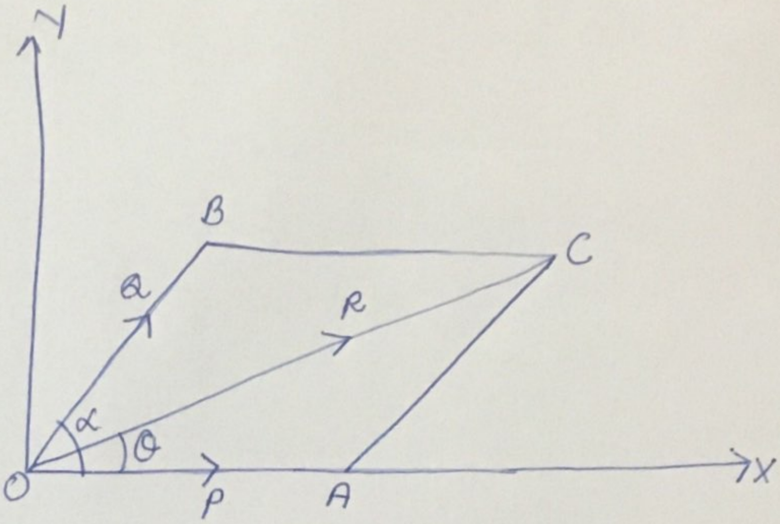

If two non-parallel forces acting on a point are represented in magnitude and direction by two adjacent sides of a parallelogram through the point of application, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.

Thus if two forces $\vec{P}$ and $\vec{Q}$ acting at a point O are represented completely by adjacent sides OA and OB respectively of the $\parallel^{gm}$ OACB, then resultant $\vec{R}$ of forces $\vec{P}$ and $\vec{Q}$ is completely represented by the diagonal OC.

**Theorem:-** The magnitude of resultant $\vec{R}$ of two concurrent forces $\vec{P}$ and $\vec{Q}$ acting at angle $\alpha$ is given by
$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$. Also if $\vec{R}$ makes an angle $\theta$ with the direction of $\vec{P}$, then

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right).$$

**Proof:-** Consider two concurrent forces $\vec{P}$ and $\vec{Q}$ represented completely by sides OA and OB of parallelogram OACB such that $\angle AOB = \alpha$




Then by parallelogram law of forces, $\vec{R}$ is completely represented by diagonal OC of the $\parallel^{gm}$ OACB.

Taking OA along the positive x-axis and a line perpendicular to it through O as y-axis, the coordinates of A, B and C are respectively $(P, 0)$, $(Q\cos\alpha, Q\sin\alpha)$ and $(R\cos\theta, R\sin\theta)$.

Now, as mid-points of OC and AB coincide,
(Give reason?)

therefore,

$$\left(\frac{0 + R\cos\theta}{2}, \frac{0 + R\sin\theta}{2}\right) = \left(\frac{P + Q\cos\alpha}{2}, \frac{0 + Q\sin\alpha}{2}\right)$$

$$\Rightarrow \quad R\cos\theta = P + Q\cos\alpha \quad \text{———} \quad (1)$$

$$R\sin\theta = Q\sin\alpha \quad \text{———} \quad (2)$$

Squaring and adding (1) and (2), we get

$$\frac{R^2(\cos^2\theta+\sin^2\theta)}{4} = \frac{1}{4}\left(P^2+Q^2\cos^2\alpha+2PQ\cos\alpha+Q^2\sin^2\alpha\right)$$

$$\Rightarrow R^2 = P^2+Q^2+2PQ\cos\alpha$$

$$R = \sqrt{P^2+Q^2+2PQ\cos\alpha} \quad \text{——— (3)}$$

which gives the magnitude of resultant $\vec{R}$.

On dividing (1) and (2), we get

$$\tan\theta = \frac{Q\sin\alpha}{P+Q\cos\alpha}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right) \quad \text{——— (4)}$$

which gives the direction of the resultant $\vec{R}$ w.r.t. $\vec{P}$.

<u>Cor. 1</u>. If $\vec{P}$ and $\vec{Q}$ act at right angle, then

$$R=\sqrt{P^2+Q^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{Q}{P}.$$

<u>Proof</u>:- If $\vec{P}$ and $\vec{Q}$ act at right angles, then $\alpha = \pi/2$.

$$\therefore \cos\alpha = \cos\frac{\pi}{2} = 0 \quad ; \quad \sin\alpha = \sin\frac{\pi}{2} = 1$$

$$\Rightarrow R = \sqrt{P^2+Q^2+2PQ\cdot 0} = \sqrt{P^2+Q^2}$$

and $$\tan\theta = \frac{Q\cdot 1}{P+Q\cdot 0} = \frac{Q}{P}$$

$$\Rightarrow \theta = \tan^{-1}(Q/P).$$

**Cor. 2** :- If $P = Q$, then $R = 2P\cos\frac{\alpha}{2}$ and $\theta = \frac{\alpha}{2}$.

**Proof** :- Taking $P = Q$ in 3 and 4, we get

$$R = \sqrt{P^2 + P^2 + 2PP\cos\alpha} = \sqrt{2P^2(1+\cos\alpha)}$$

$$= \sqrt{2P^2 \cdot 2\cos^2\frac{\alpha}{2}} = 2P\cos\left(\frac{\alpha}{2}\right)$$

and
$$\tan\theta = \frac{P\sin\alpha}{P + P\cos\alpha} = \frac{2P\sin\alpha/2\,\cos\alpha/2}{P(1+\cos\alpha)}$$

$$= \frac{2\sin\alpha/2\,\cos\alpha/2}{2\cos^2\alpha/2} = \tan\frac{\alpha}{2}$$

$$\Rightarrow \theta = \alpha/2$$

Thus, resultant of two forces equal in magnitude bisects the angle between the forces.